Intrinsic and Extrinsic Noise Effects on Molecular Distributions in Bacteria

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Central dogma



Deterministic inducible regulation



Dependence of bistability on κ_d and K



Intrinsic and Extrinsic noise



Bursting (intrinsic noise) in cells

- Experimentally observed that in many organisms the amplitude of protein production through bursting translation of mRNA is exponentially distributed at the single cell level
- Let the density of this distribution be $h(y) = \frac{1}{b}e^{-y/b}$



Yu (2006), Science, 311

Bursting as a jump Markov process

Replace the simple deterministic dynamics

$$\frac{dx}{dt} = -\gamma x + \gamma \kappa_d f(x)$$

with

$$\frac{dx}{dt} = -\gamma x + \Xi(h, \gamma \kappa_b f(x))$$

where $\Xi(h, \varphi)$ is a jump Markov process occurring at a rate φ and distributed with density h

Stationary density as a function of κ_b





Effects of bursting



Noise induced by bursting



Extrinsic noise as a white noise process

Replace the simple deterministic dynamics

$$\frac{dx}{dt} = -\gamma x + \gamma \kappa_d f(x)$$

with

$$dx = [\gamma \kappa_d f(x) - \gamma x]dt + \sigma \sqrt{x}dw$$

where dw is a standard white noise process

Extrinsic noise \sim intrinsic noise

• Let replace the average burst amplitude *b* with $b \rightarrow \sigma^2/2\gamma \equiv b_w$ and $\kappa_b \rightarrow \kappa_e = 2\gamma\kappa_d/\sigma^2 \equiv \kappa_d/b_w$, then the stationary density in the case of extrinsic noise has the same form as in the case of intrinsic noise and the same results hold.



Conclusion

- Intrinsic noise and extrinsic noise are indistinguishable from the stationnary density in this model.
- The stationary densities can be much more wider (and asymmetric) than a poissonian ditribution.
- Noise-enhanced bistability.
- And noise-induced bistability when n = 1.
- When both noise are present, their effect sum up additively.

Problems and Further Studies

- Intrinsic noise should take into account transcriptional and/or translational delays.
- Extrinsic noise should take into account time correlations.
- Mean exit time and auto-correlation function should be derived to give more information on the dynamics out of equilibirum.
- The full three-stage model should be considered with noise.

Some References (Not exhaustive)

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